

Rencontre ANR Fatou – du 16 au 18 avril 2018

LUNDI 16 AVRIL 2018

**10h:** Accueil des participants - café

**10h30-12h:** FABRIZIO BIANCHI (Imperial College): **Bifurcation of polynomial-like maps in higher dimension I**

Pause déjeuner

**14h-15h:** MATTEO RUGGIERO (Université Paris-Diderot): **Super currents and super forms**

Pause café

**15h30-17h:** JUNYI XIE (Université CNRS, Université Rennes 1): **Algebraic dynamics of polynomial endomorphisms of the affine plane I**

MARDI 17 AVRIL 2018

**9h30-10h30:** FABRIZIO BIANCHI (Imperial College): **Bifurcation of polynomial-like maps in higher dimension II**

**10h30-12h:** MATTHIEU ASTORG (Université d'Orléans): **Bifurcations in families of polynomial skew products**

Pause déjeuner

**14h-15h:** HAN PETERS (University of Amsterdam): **Zeros of the Ising partition function**

Pause café

**15h30-16h30:** JUNYI XIE (Université CNRS, Université Rennes 1): **Algebraic dynamics of polynomial endomorphisms of the affine plane II**

MERCREDI 18 AVRIL 2018

**9h-10h:** JOHAN TAFLIN (Université de Bourgogne): **Blenders near polynomial product maps of  $\mathbb{C}^2$**

**10h30-11h30:** SÉBASTIEN BIEBLER (Université Paris-Est Marne-la-vallée): **Lattès maps and the interior of the bifurcation locus**

## RÉSUMÉS

### MATTHIEU ASTORG: **Bifurcations in families of polynomial skew products**

This is a joint work with Fabrizio Bianchi.

We investigate the parameter space of the family of polynomial skew-products, especially the family of quadratic skew-products over a given base  $p$ , that is, maps of the form

$$F(z, w) = (p(z), q(z, w)),$$

where  $p$  and  $q$  are quadratic polynomials and  $p$  is fixed. We describe the geometry of the bifurcation current near infinity, and we give a partial classification of hyperbolic components. One of the tools we use is the equidistribution of some dynamically defined hypersurfaces in parameter space towards the bifurcation current, analogously to the dimension one case.

### FABRIZIO BIANCHI: **Bifurcation of polynomial-like maps in higher dimension I + II**

In these talks we discuss stability of holomorphic dynamical systems under perturbation. In dimension 1, the theory is now classical and is based on works by Lyubich, Mané-Sad-Sullivan and DeMarco. I will quickly review this theory and present a recent generalisation valid in any dimension, in the setting of polynomial-like maps of large topological degree. Since classical 1-dimensional techniques no longer apply in higher dimensions, our approach is based on ergodic and pluripotential methods.

- (1) Setting, statement, and general strategy,
- (2) Holomorphic motions in several variables and Misiurewicz parameters.

### SEBASTIEN BIEBLER: **Lattès maps and the interior of the bifurcation locus**

Je montrerai que pour un tore complexe bidimensionnel donné, il existe un entier  $d$  tel que tout exemple de Lattès de degré  $> d$  qui induit une application affine sur ce tore est dans l'adhérence de l'intérieur du lieu de bifurcation. En particulier, tout exemple de Lattès possède un itéré accumulé par des bifurcations robustes. Le principe général pour obtenir de telles bifurcations robustes réside en l'obtention d'intersections persistantes entre l'ensemble de Julia de l'application perturbée et son ensemble postcritique. J'introduirai tout d'abord un toy-model permettant d'obtenir des intersections persistantes entre un certain IFS ayant des propriétés auto-correctrices et une courbe. Ensuite, je montrerai qu'il est possible de perturber un exemple de Lattès de sorte à se ramener à ce toy-model.

### HAN PETERS: **Zeros of the Ising partition function**

This is a work in progress with Guus Regts.

The Ising model originated in statistical physics, where it was used to model magnetic objects. Absence of zeros of the partition function implies the absence of phase transitions, i.e. it implies analytic dependence on the parameters, a classical result of Lee and Yang. More recently there has been considerable interest from

the perspective of algorithmic complexity. In complexity theory, absence of zeros implies the existence of fast approximation algorithms.

The partition function of a graph depends on two parameters, a field-like parameter  $\lambda$ , and a temperature-like parameter  $\beta$ . A result of Lee-Yang from 1952 states that for the physically relevant  $\beta \in [-1, 1]$ , the partition function can only be zero when  $\lambda$  lies on the unit circle. The famous Lee-Yang Theorem has received enormous attention in the literature. The exact  $\lambda$ -values for which the partition function is always non-zero has been studied, but not completely described. We will give a complete description for arbitrary graphs of maximal degree  $d \geq 2$ . Our result is obtained by reducing the problem to the setting of trees, where it can be studied using classical techniques from complex dynamical systems.

**MATTEO RUGGIERO: Super currents and super forms**

Super forms and currents are introduced as a real analogous of (p,q)-forms and currents of the complex setting. We show how tropical spaces can be interpreted as supports of (suitable) positive super currents.

**JOHAN TAFLIN: Blenders near polynomial product maps of  $\mathbb{C}^2$**

We will explain how a construction coming from smooth dynamics, called blenders, gives rise to open sets of endomorphisms with specific phenomena as robust bifurcations, attracting sets with non-empty interior or saddle point with dense unstable manifold.

**JUNYI XIE: Algebraic dynamics of polynomial endomorphisms of the affine plane I + II**

The valuation tree is the space of normalized valuations on  $\mathbb{C}[x, y]$  at the infinity. It is introduced by Favre and Jonsson. In these two talks, I will induce the theory of the valuation tree and apply it to some problems in polynomial dynamics on the affine plane.