The Dade group of a finite group and dimension functions

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In this talk, I will present some results from our recent joint work with Matthew Gelvin [1]. Let G be a finite group and k an algebraically closed field of characteristic $p > 0$. In this work, we define the notion of a Dade kG-module as a generalization of endo-permutation modules for p-groups. We show that under a suitable equivalence relation, the set of equivalence classes of Dade kG-modules forms a group under tensor product, and the group obtained this way is isomorphic to the Dade group $D(G)$ defined by Lassueur [2].

We also consider the subgroup $D^{\Omega}(G)$ of $D(G)$ generated by relative syzygies Ω_X, where X is a finite G-set. Let $C(G, p)$ denote the group of superclass functions defined on the p-subgroups of G. There are natural generators ω_X of $C(G, p)$. We prove that there is a well-defined group homomorphism $\Psi_G : C(G, p) \to D^{\Omega}(G)$ that sends ω_X to Ω_X.

The main theorem of this work is the verification that the subgroup of $C(G, p)$ consisting of the dimension functions of k-orientable real representations of G lies in the kernel of Ψ_G. In the proof we consider Moore G-spaces which are the equivariant versions of spaces which have nonzero reduced homology in only one dimension.

References

[1] M. Gelvin and E. Yalçın, Dade Groups for Finite Groups and Dimension Functions, preprint, 2020 (arXiv:2007.05322v2).

[2] C. Lassueur, *The Dade group of a finite group*, J. Pure Appl. Algebra, **217** (2013), 97-113.